A Two-dimensional Elemental Operator for Vectorial Hysteresis Model of Magnetic Material

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This paper employs a two-dimensional elemental operator with biaxial anisotropy based on the physical mechanisms of the cubic textured magnetic materials, and derives an improved analytical expression of the direct relationship between *H* and *M* for a single elemental operator by taking into account the ineraction field and the overall anisotropy coefficient. The magnetic properties of this two-dimensional elemental operator under altnating magnetic field and rotating magnetic field have been analyzed. Meanwhile, to model the macroscopic magnetic hysteresis of a specimen, this approach utilizes the concept of distribution function density in Preisach model in order to consider the interaction field and the coercive force of each elemental operator. The vectorial elemental operator in this study is derived in the form of two dimensions, and adopted to model the scalar magnetic hysteresis by restricting the applied magnetic field to vary along one dimension.

***Index Terms*—Two-dimensional elemental operator, vectorial hysteresis model, magnetic material.**

I. Introduction

VARIOUS hysteresis models have been proposed based on the superposition of lots of hysteresis operators, such as the Preisach model, the Stoner-Wohlfarth (S-W) model, and the Della Torre-Pinzaglia-Cardelli (DPC) model [1]. The operator in the scalar Preisach model is a rectangular hysteron, which only has two directions (±1), thus difficult to describe the vectorial relationship between *H* and *M* [2]. The S-W model based on the vectorial S-W particle with uniaxial anisotropy is inconsistent with the physical mechanisms of magnetic materials [3]. The phenomenological vector hysteresis operator (hysteron) in DPC model has weak appeal to physical intuition [4]. Therefore, the vectorial properties of magnetic material have not been successfully described and analyzed, not to mention the application for engineering practice.

In this paper, a two-dimensional elemental operator is introduced to establish the vectorial hysteresis model of magnetic material. Then, the magnetic properties, especially the complex rotational magnetic properties of the elemental operator, have been analyzed.

II. Model Description

In this paper, it is assumed that the magnetic material is composed of interacting elemental operator which possesses its own biaxial anisotropy and anisotropy coefficient as illusrtrited in Fig. 1. Then, the magnetization direction of each biaxial elemental operator is determined from the various energy contributions of the material anisotropy and applied field. Thus, the energy of a biaxial elemental operator can be written as

$$E=K\sin^2\theta\cos^2\theta-\mu_0 HM\cos(\theta_H-\theta) \qquad (1)$$

where θ_H and θ are the angles of the applied field *H* and the resultant magnetization, respectively, both with respect to the particle's anisotropic (easy) axis, *K* is the anisotropy coefficient, and *M* the magnetization. The first term represents the biaxial anisotropic energy, and the second the interaction energy associated with the external magnetic field and the particle magnetization.

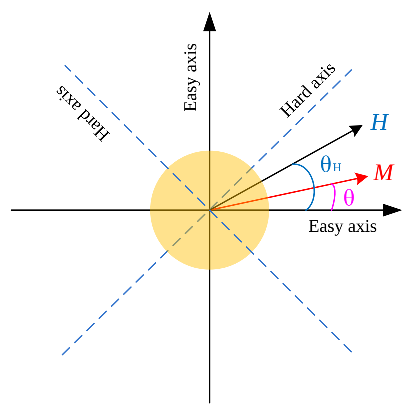

Fig. 1 Two-dimensional elemental operator with two easy axes and two hard axes.

For different applied field exerted on the operator, the total energy and the corresponding energy minimum will be different, as shown in Fig. 2. However, it is difficult to directly obtain the minimum for there is no analytic solution for this high-order energy equation.

Similar to the determination method of S-W particle, the stable orientation of the magnetization vector for one elemental operator can be obtained by geometry. The interface of energy maxima and energy minima can be determined from

$$\frac{\partial E}{\partial\theta}=\frac{1}{2}K\sin 4\theta-\mu_0 HM_S\sin(\theta_H-\theta)=0 \qquad (2)$$

$$\frac{\partial^2 E}{\partial \theta^2} = 2K\cos 4\theta + \mu_0 H M_S \cos(\theta_H - \theta) = 0 \tag{3}$$

Thus,

$$\mathrm{H}_x = \frac{2K}{\mu_0 \mathrm{m}} \cos^3\theta (5 - 6\cos^2\theta) \tag{4}$$

$$\mathrm{H}_y = \frac{2K}{\mu_0 \mathrm{m}} \sin^3\theta (5 - 6\sin^2\theta) \tag{5}$$

where $\mathrm{H}_x = H\cos\theta$ and $\mathrm{H}_y = H\sin\theta$, respectively. And this equations represent octagonal curve, as shown in Fig. 3(a).

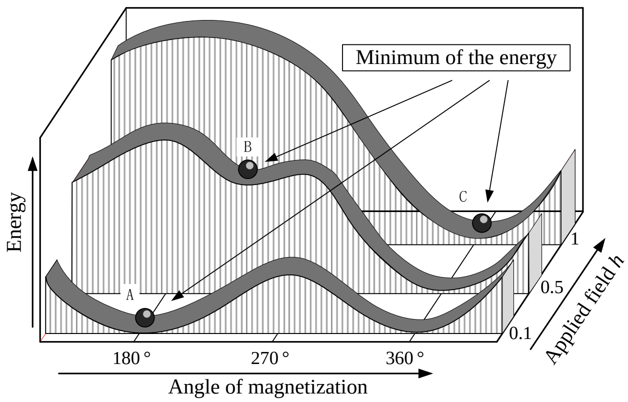

Fig. 2. Energy oscillogram of one elemental operator under different applied fields.

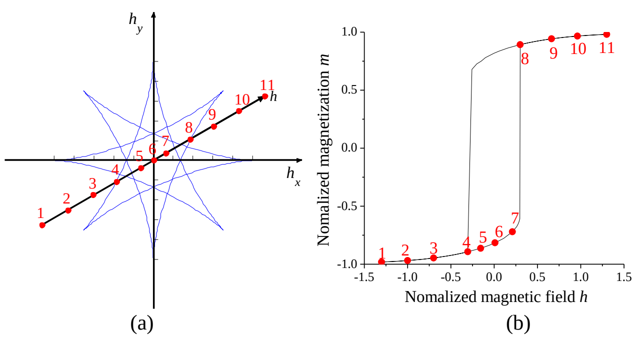

Fig. 3. (a) The increasing field applies on the octagonal curve and (b) the corresponding points on the hysteresis curve.

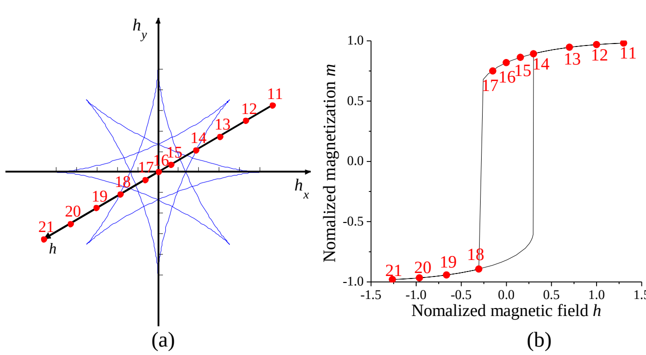

Fig. 4. (a) The decreasing field applies on the octagonal curve and (b) the corresponding points on the hysteresis curve.

When the magnetic field increasing on the elemental operator, as shown in Fig. 3(a), from point 1 to point 11, the corresponding magnetization can compose parts of the hysteresis loop, as shown in Fig. 3(b). Meanwhile, when the magnetic field decreasing on the elemental operator, from 11 to 21, the corresponding properties can be illustrated in Fig. 4. Note that, if this elemental operator under alternating field, Fig. 3(b) and Fig. 4(b) can compose an integrated hysteresis loop.

If the rotational magnetic field H applied on the elemental operator, the orientation of the magnetization M will lag behind or lead magnetic field, as shown in Fig. 5. For different magnitude of the rotational magnetic field H, the loci of the magnetization M will be different.

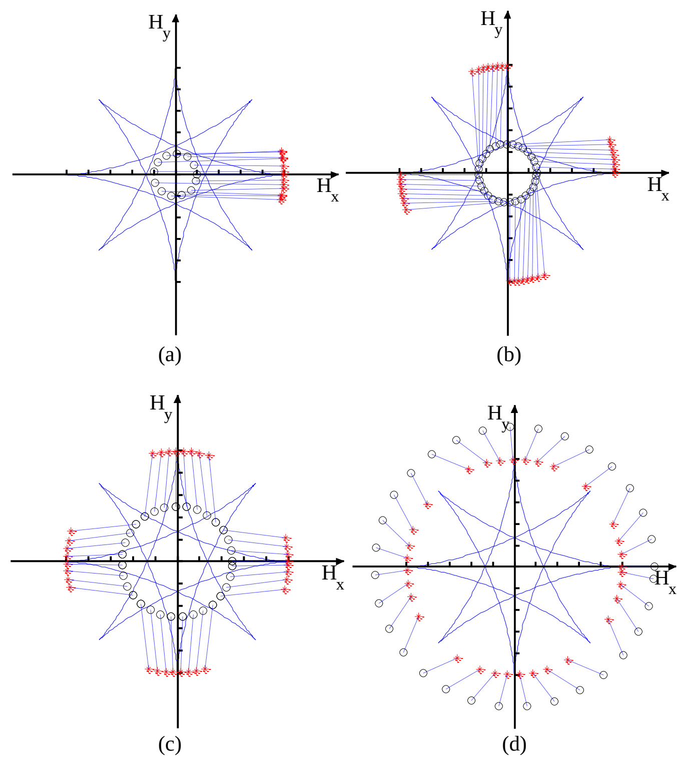

Fig. 5. The rotational magnetic properties between applied field (dark circle) and the corresponding magnetization (red star).

III. Experimental Verification and Conclusion

To verify the proposed two-dimensional elemental operator and the corresponding vectorial hysteresis model, the magnetic hysteresis of soft magnetic composite material and a non-oriented silicon steel Lycore-140 sample under alternating excitations and rotating excitations has been simulated and compared with the experimental results. The good agreement illustrates the validity and practicability of this two-dimensional elemental operator.

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